The Game within the Game: Rating NFL offenses and defenses

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Wartburg Math, Computer Science, and Physics Department Seminar
November 25, 2025

UNIVERSITY OF WISCONSIN
PLATTEVILLE
Department of Mathematics

Thank you!

Thanks for being here, and for inviting me — it's a pleasure to be here!

NCAA Division III football playoff

2025 DIVISION III PLAYOFF BRACKET

| First round Nov. 22 | Second Round Nov. 29 | Third Round Dec. 6 | Quarterfinal Dec. 13 | Semifinal Dec. 20 | Stagg Bowl LII Jan. 4, 2026 | Semifinal Dec. 20 | Quarterfinal Dec. 13 | Third round Dec. 6 | Second round Nov. 29 | First round Nov. 22 |

at Mount Union (10-0)

at Muhlenberg (8-2)

Muhlenberg, 34-26

Union (8-2)

at John Carroll (9-1)

Randolph-Macon (9-1)

at Trinity, Texas (9-1)

Hardin-Simmons (9-1)

LaGrange (9-1)

LaGrange, 24-21

at Framingham State (7-3)

at Berry (9-1)

at North Central (10-0)

at Hanover (8-2)

Hanover, 23-15

Grove City (8-2)

at Hope (9-1)

UW-La Crosse (7-2)

at UW-Platteville (8-2)

Alma (8-2)

at Concordia Wisconsin (9-1)

Coe, 44-7

Coe (8-2)

at Bethel (10-0)

NATIONAL CHAMPION
(Canton, Ohio, 8 p.m. ET)

at Wartburg (10-0)

at Wheaton (8-2)

Wheaton, 76-14

Crown (7-3)

at UW-Whitewater (8-2)

DePauw (9-1)

at St. John's (9-1)

Monmouth (9-1)

at Whitworth (8-2)

Chapman, 18-16

Chapman (8-2)

at UW-River Falls (9-1)

at Christopher Newport (10-0)

at Susquehanna (8-2)

Susquehanna, 36-32

Washington & Jefferson (8-2)

at Franklin & Marshall (9-1)

Eastern (9-1)

at Salisbury (9-1)

Endicott (9-1)

at Springfield (8-2)

Springfield, 21-7

Cortland (8-2)

at Johns Hopkins (9-1)

Get full Division III playoff coverage at D3football.com!

NCAA Division III football playoff

D3football.com Top 25, 2025 Week 11

Through games of Nov. 15:

- Previous years' Top 25 polls

Rank	School (No. 1 votes)	W-L	Points	Prev.
1	North Central (Ill.) (25)	10-0	625	1
2	Mount Union	10-0	595	2
3	UW-River Falls	9-1	572	4
4	Bethel	10-0	539	5
5	Wartburg	10-0	533	6
6	Christopher Newport	10-0	471	7
7	St. John's	9-1	440	8
8	Johns Hopkins	9-1	430	3
9	John Carroll	9-1	414	9
10	UW-Platteville	8-2	389	10
11	UW-Whitewater	8-2	342	12
12	Salisbury	9-1	326	11
13	UW-La Crosse	7-2	322	14
14	Hardin-Simmons	9-1	319	13
15	DePauw	9-1	262	15
16	Franklin and Marshall	9-1	254	25
17	Berry	9-1	227	17
18	Hope	9-1	225	16
19	Wheaton (Ill.)	8-2	211	18
20	Randolph-Macon	9-1	160	21
21	Trinity (Texas)	9-1	110	22
22	Susquehanna	8-2	87	23
23	Alma	8-2	52	--
24	Coe	8-2	38	--
25	Monmouth	9-1	25	--

Dropped out: No. 19 Central; No. 20 Cortland; No. 24 Wabash.

Others receiving votes: Mary Hardin-Baylor 23; Central (Iowa) 21; Muhlenberg 19; Endicott 19; Cortland 17; Washington & Jefferson 15; Chapman 13; Wabash 7; Eastern 5; Whitworth 4; Baldwin Wallace 4; Grove City 3; Rowan 3; Hanover 2; Ithaca 2.
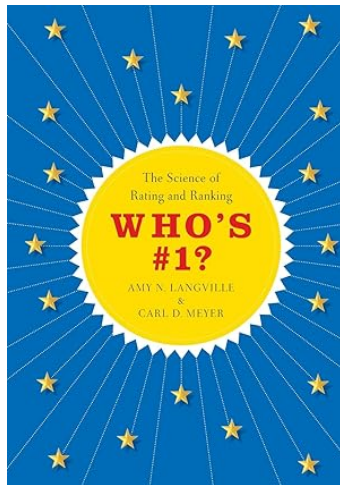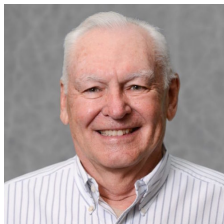
Methods for rating teams
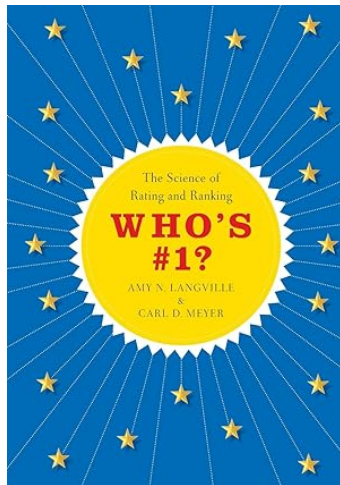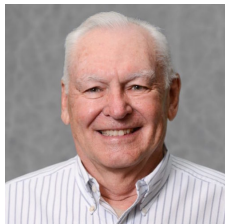
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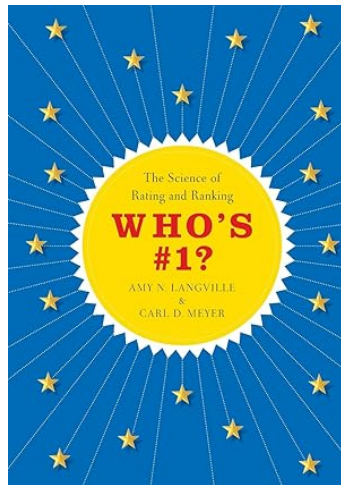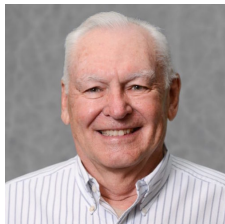
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The Science of
Rating and Ranking

WHO'S #1?

AMY N. LANGVILLE
&
CARL D. MEYER

Methods for rating teams

- Many rating methods exist [6]
- Different hypotheses
- Goal: Evaluate or predict?
  - margin of victory
  - home/away
  - recency
  - injuries

The Science of
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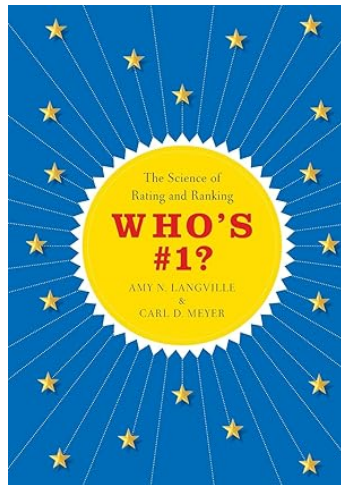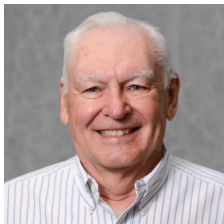
Prof. Kenneth Massey

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- **model hypothesis:**
  margin of victory = difference in team ratings

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Massey ratings [8]
- honors project, 1997
- BCS component, 1999–2013
- **model hypothesis:**
  margin of victory = difference in team ratings
  - Knights 19, Pioneers 14 ⇒ rate Wartburg 5 points higher than UWP

A slight problem with the Massey ratings?

ARC 2025: selected results [1]

Dubuque 34, Loras 27	$D - L = 7$
Central 35, Dubuque 16	$C - D = 19$
Central 45, Loras 27	$C - L = 18$
Wartburg 33, Dubuque 7	$W - D = 26$
Wartburg 40, Loras 20	$W - L = 20$
Wartburg 28, Central 13	$W - C = 15$

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$$0 = (D - L) + (C - D) - (C - L)$$

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$$0 = (D - L) + (C - D) - (C - L) = 7 + 19 - 18 = 8$$

Massey

ARC 2025

$$D - L = 7$$
$$C - D = 19$$
$$C - L = 18$$
$$W - D = 26$$
$$W - L = 20$$
$$W - C = 15$$

Massey in matrices

ARC 2025

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Matrix form

$$\begin{bmatrix} 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C \\ D \\ L \\ W \end{bmatrix} = \begin{bmatrix} 7 \\ 19 \\ 18 \\ 26 \\ 20 \\ 15 \end{bmatrix}$$

$$\boxed{X\vec{r} = \vec{y}}$$

Statistics to the rescue

- No $\vec{r}$ solves $X\vec{r} = \vec{y}$

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$$
\begin{bmatrix}
0 & 1 & 1 & 0 & 0 & -1 \\
1 & -1 & 0 & -1 & 0 & 0 \\
-1 & 0 & -1 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
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C \\ D \\ L \\ W
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$(C, D, L, W) = (5.5, -9.5, -11.25, 15.25)$

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New model hypothesis: Rate each team's offense and defense.

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Massey's offense/defense ratings

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Twice as many variables, twice as many equations!

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ARC 2025

Offense		Defense			
Central	30.2	Wartburg	13.0		
Wartburg	29.3	Central	2.4		
Loras	29.1	Dubuque	−2.1		
Dubuque	19.7	Loras	−13.3		

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ARC 2025

Offense		Defense		Offense + Defense	
Central	30.2	Wartburg	13.0	Wartburg	42.3
Wartburg	29.3	Central	2.4	Central	32.6
Loras	29.1	Dubuque	−2.1	Dubuque	17.6
Dubuque	19.7	Loras	−13.3	Loras	15.8

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Wartburg	29.3	Central	2.4	Central	32.6	5.5
Loras	29.1	Dubuque	−2.1	Dubuque	17.6	− 9.5
Dubuque	19.7	Loras	−13.3	Loras	15.8	− 11.25

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- Change to 50/50?

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Central	16.7	Wartburg	26.5	Wartburg	42.3
Wartburg	15.8	Central	15.9	Central	32.6
Loras	15.5	Dubuque	11.4	Dubuque	17.6
Dubuque	6.2	Loras	0.3	Loras	15.8

Comparing offenses to defenses

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- Change to 50/50?
- . . . and translate average to 0?

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Wartburg	2.3	Central	2.4	Central	5.5
Loras	2.0	Dubuque	−2.1	Dubuque	−9.5
Dubuque	−7.4	Loras	−13.3	Loras	−11.3

Implementation

Scaling the method

- Rating 4 teams: feasible by hand
- 32 teams: automate!

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Scaling the method

- Rating 4 teams: feasible by hand
- 32 teams: automate!
  - Accurate, *uniform* data is vital.

Implementation

```python
def original_massey(games_between, points_for, points_against, teams):
    label = []   # list of units
    unit_rating = dict()
    team_count = 0
    index = dict()   # index[x] holds position of unit x in label
    for x in teams:
        index[x]=team_count
        label.append(x)
        team_count += 1

    P = np.zeros((team_count, team_count), dtype=int)
    T = np.zeros((team_count, team_count), dtype=int)
    f = np.zeros(team_count)
    a = np.zeros(team_count)
    for pair in games_between: # pair looks like ('MIN', 'GB'), and appears in both orders
        P[index[pair[0]]][index[pair[1]]] += len(games_between[pair])  # number of pairwise matchups
        T[index[pair[0]]][index[pair[0]]] += len(games_between[pair])  # number of games played
    for team in teams:
        f[index[team]] += points_for[team]
        a[index[team]] += points_against[team]

    mat = T-P
    point_diff = f-a
    mat[-1] = np.ones(team_count)  # create unique solution while forcing sum of ratings to 0
    point_diff[-1] = 0

    rating = np.linalg.solve(mat, point_diff)
    rating_def = np.linalg.solve(T+P, T.dot(rating)-f)      # see Who's #1, p. 12
    rating_off = rating-rating_def

    for team in teams:
        unit_rating[team+'_off'] = rating_off[index[team]]
        unit_rating[team+'_def'] = rating_def[index[team]]
    return unit_rating, label, teams, mat, point_diff
```

Midwest Sports Analytics Meeting

2018 Keynote Address: Tim Chartier

Midwest Sports Analytics Meeting

Midwest Sports Analytics Meeting

The Game within the Game

MLB player ratings [5], [12]

- Chartier mentioned a project done at Furman University:
  - Prof. John Harris
  - Prof. Kevin Hutson
  - Will Decker
  - Jordan Lyerly
  - Aaron Markham
  - Rob Picardi

The Game within the Game

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- Treat each *player* as a team! Each *at-bat* is its own game, pitcher vs. batter.

The Game within the Game

"Subgames" in the NFL

- Car ride home with Ben Collins (Epic Systems): The football analogue of an at-bat is a **drive**, and the analogue of a batter (pitcher) is an offense (defense).

The Game within the Game

"Subgames" in the NFL

- Car ride home with Ben Collins (Epic Systems): The football analogue of an at-bat is a **drive**, and the analogue of a batter (pitcher) is an offense (defense).
- With Dan Swenson (Black Hills State University): Turn this idea into computations, and figure out how to evaluate it!

Expected points and team ratings

Football: evaluating a drive [11]

Teams could, for example, be judged by how they perform relative to expectation. . . . If the offensive team begins at their 25-yard line and scores a field goal then they have earned 3 points, 2.76 more than might have been expected at the start of the possession. The contributions of the offense, defense, and special teams could be measured separately.

— Hal S. Stern

Implementation

2025 NFL ratings (through Week 11*)

Implementation

- 64 "teams," 3149 "games": Computers are necessary!

Implementation

2025 NFL ratings (through Week 11*)

- 64 "teams," 3149 "games": Computers are necessary!
- Accurate, *uniform* data is vital.
  - Play-by-play data: `nflfastR` [2]

	week	posteam	defteam	drive	qtr	down	time	yrdln	ydstogo	desc
1	week	posteam	defteam	drive	qtr	down	time	yrdln	ydstogo	desc
2	1				1		15:00	NO 35		0 GAME
3	1	ARI	NO	1	1		15:00	NO 35		0 19-B.Grupe kicks 65 yards from NO 35 to ARI 0. 4-G.Dortch to ARI 22 for 22 yards (28-D.Stutsman).
4	1	ARI	NO	1	1	1	14:56	ARI 22	10	(14:56) 6-J.Conner right tackle to ARI 25 for 3 yards (92-D.Godchaux).
5	1	ARI	NO	1	1	2	14:18	ARI 25	7	(14:18) (Shotgun) 1-K.Murray pass short left to 85-T.McBride pushed ob at ARI 36 for 11 yards (27-I.Yiadom).
6	1	ARI	NO	1	1	1	13:40	ARI 36	10	(13:40) 1-K.Murray sacked at ARI 25 for -11 yards (94-C.Jordan).
7	1	ARI	NO	1	1	2	12:57	ARI 25	21	(12:57) (Shotgun) 6-J.Conner left end to ARI 23 for -2 yards (4-K.McKinstry).
8	1	ARI	NO	1	1	3	12:14	ARI 23	23	(12:14) (Shotgun) 33-T.Benson left guard to ARI 24 for 1 yard (56-D.Davis).
9	1	ARI	NO	1	1	4	11:39	ARI 24	22	(11:39) 12-B.Gillikin punts 60 yards to NO 16, Center-46-A.Brewer. 22-R.Shaheed to NO 23 for 7 yards (27-A.Davis-Gaither; 84-E.Higgins).
10	1	NO	ARI	2	1		11:27	NO 23	10	(11:27) (Shotgun) 41-A.Kamara up the middle to NO 26 for 3 yards (27-A.Davis-Gaither; 93-C.Campbell).
11	1	NO	ARI	2	1	2	10:55	NO 26	7	(10:55) (Shotgun) 2-S.Rattler pass incomplete short middle to 22-R.Shaheed (0-W.Johnson).
12	1	NO	ARI	2	1	3	10:51	NO 26	7	(10:51) (Shotgun) 2-S.Rattler pass incomplete short right.
13	1	NO	ARI	2	1	4	10:46	NO 26	7	(10:46) 32-K.Kroeger punts 47 yards to ARI 27, Center-49-Z.Wood. 4-G.Dortch to ARI 34 for 7 yards (58-C.Rumph; 53-J.Ford).
14	1	ARI	NO	3	1	1	10:34	ARI 34	10	(10:34) 1-K.Murray pass short left to 85-T.McBride to ARI 39 for 5 yards (21-J.Reid).
15	1	ARI	NO	3	1	2	10:06	ARI 39	5	(10:06) (No Huddle, Shotgun) 1-K.Murray scrambles right end ran ob at NO 48 for 13 yards (21-J.Reid).
16	1	ARI	NO	3	1	1	9:28	NO 48	10	(9:28) (No Huddle, Shotgun) 6-J.Conner left tackle to NO 47 for 1 yard (23-J.Blackmon).
17	1	ARI	NO	3	1	2	8:55	NO 47	9	(8:55) (No Huddle) 1-K.Murray sacked ob at NO 47 for 0 yards (96-C.Granderson).
18	1	ARI	NO	3	1	3	8:11	NO 47	9	(8:11) (Shotgun) 1-K.Murray pass incomplete short middle to 18-M.Harrison (4-K.McKinstry). PENALTY on NO-4-K.McKinstry, Defensive Pass Inte
19	1	ARI	NO	3	1	1	8:05	NO 34	10	(8:05) (Shotgun) PENALTY on ARI-74-I.Adams, False Start, 5 yards, enforced at NO 34 - No Play.
20	1	ARI	NO	3	1	1	8:05	NO 39	15	(8:05) (Shotgun) 1-K.Murray sacked at NO 39 for 0 yards (20-P.Werner).
21	1	ARI	NO	3	1	2	7:22	NO 39	15	(7:22) (Shotgun) 6-J.Conner up the middle to NO 27 for 12 yards (21-J.Reid; 58-C.Rumph).
22	1	ARI	NO	3	1	3	6:38	NO 27	3	(6:38) (Shotgun) 1-K.Murray pass short right to 14-M.Wilson to NO 22 for 5 yards (20-P.Werner).
23	1	ARI	NO	3	1	1	5:58	NO 22	10	(5:58) 6-J.Conner up the middle to NO 19 for 3 yards (56-D.Davis).

Implementation

2025 NFL ratings (through Week 11*)

- 64 "teams," 3149 "games": Computers are necessary!
- Accurate, *uniform* data is vital.
  - ▹ Play-by-play data: `nflfastR` [2]

```
Description
Drive: ARI vs. NO wk 1 (Punt --- 7 plays from NO 35, 2 yards, 0 points, -2.39 EPA, -4.66% WPA)
Drive: NO vs. ARI wk 1 (Punt --- 4 plays from NO 23, 3 yards, 0 points, -2.97 EPA, -6.20% WPA)
Drive: ARI vs. NO wk 1 (Field goal --- 14 plays from ARI 34, 42 yards, 3 points, 1.42 EPA, 4.03% WPA)
Drive: NO vs. ARI wk 1 (Touchdown --- 16 plays from ARI 35, 75 yards, 7 points, 5.36 EPA, 17.60% WPA)
Drive: ARI vs. NO wk 1 (Touchdown --- 10 plays from NO 35, 65 yards, 7 points, 5.88 EPA, 18.87% WPA)
Drive: NO vs. ARI wk 1 (Field goal --- 12 plays from ARI 35, 37 yards, 3 points, 1.49 EPA, 5.80% WPA)
Drive: ARI vs. NO wk 1 (Touchdown --- 14 plays from NO 35, 71 yards, 7 points, 6.03 EPA, 22.95% WPA)
Drive: NO vs. ARI wk 1 (End of half --- 2 plays from ARI 35, 6 yards, 0 points, -0.60 EPA, -3.39% WPA)
Drive: NO vs. ARI wk 1 (Punt --- 7 plays from ARI 35, 16 yards, 0 points, -2.48 EPA, -0.91% WPA)
Drive: ARI vs. NO wk 1 (Field goal --- 6 plays from ARI 19, 49 yards, 3 points, 2.36 EPA, 7.31% WPA)
Drive: NO vs. ARI wk 1 (Missed FG --- 17 plays from ARI 35, 53 yards, 0 points, -2.86 EPA, -5.49% WPA)
Drive: ARI vs. NO wk 1 (Punt --- 6 plays from ARI 28, 11 yards, 0 points, -2.88 EPA, -1.43% WPA)
Drive: NO vs. ARI wk 1 (Punt --- 7 plays from NO 27, 13 yards, 0 points, -1.68 EPA, -4.17% WPA)
Drive: ARI vs. NO wk 1 (Punt --- 11 plays from ARI 6, 35 yards, 0 points, -0.85 EPA, 1.62% WPA)
Drive: NO vs. ARI wk 1 (Punt --- 5 plays from NO 17, -5 yards, 0 points, -4.24 EPA, -6.31% WPA)
Drive: ARI vs. NO wk 1 (Missed FG --- 8 plays from NO 40, 12 yards, 0 points, -5.09 EPA, -2.54% WPA)
Drive: NO vs. ARI wk 1 (Field goal --- 10 plays from NO 36, 54 yards, 3 points, 1.28 EPA, -0.40% WPA)
```

2025 NFL unit rankings — numerically

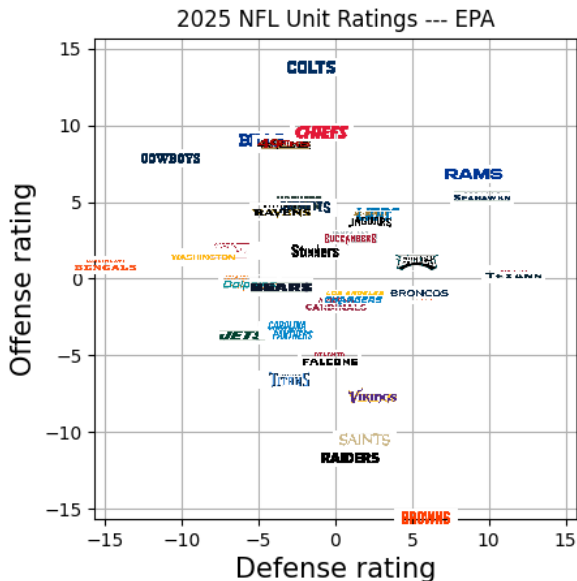| Team ratings	
LA	15.9
SEA	14.9
IND	12.1
HOU	11.9
KC	8.7
⋮	
TEN	-9.7
CLE	-9.7
NYJ	-9.8
LV	-10.7
CIN	-14.2

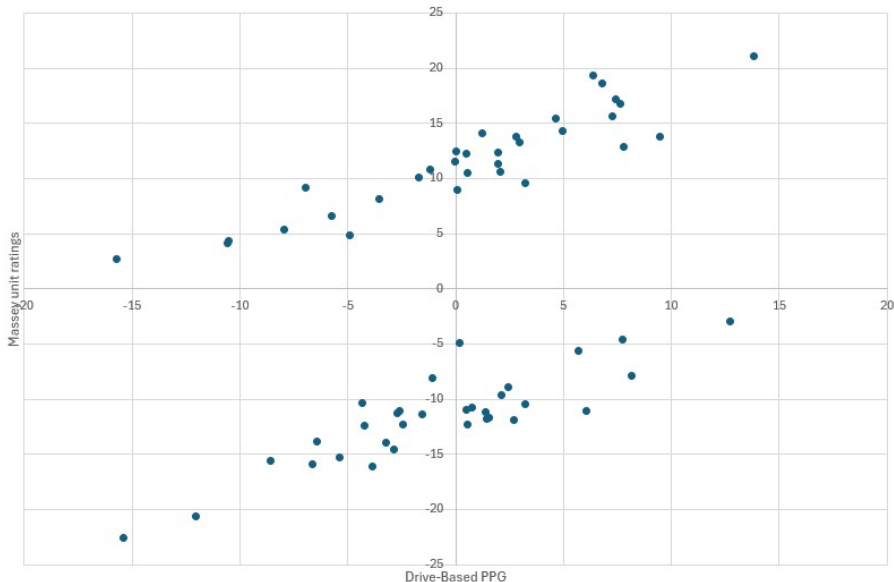| Offenses	
IND	13.8
KC	9.6
BUF	9.0
SF	8.7
DAL	7.8
⋮	
TEN	-6.6
MIN	-7.7
NO	-10.6
LV	-11.7
CLE	-15.6

Defenses	
HOU	11.6
SEA	9.6
LA	9.1
CLE	5.9
DEN	5.4
⋮	
NYJ	-6.1
NYG	-6.8
WAS	-8.6
DAL	-10.8
CIN	-15.0

2025 NFL unit rankings — graphically

2025 NFL Unit Ratings --- EPA

Cross-validation with Massey's unit rankings

Original Massey unit PPG vs. Drive-based unit PPG

Predictive power?

How do you evaluate predictions?

Predictive power?

How do you evaluate predictions?

- Bet a lot of money?

Predictive power?

How do you evaluate predictions?

- Bet a lot of money?
- Count correct predictions?

Predictive power?

How do you evaluate predictions?

- Bet a lot of money?
- Count correct predictions?
- Should predictions be all-or-nothing?

Brier score

Evaluating probabilistic predictions

- Brier [3]: Score probabilistic forecasts by mean squared error

Glenn W. Brier

Brier score

Evaluating probabilistic predictions

- Brier [3]: Score probabilistic forecasts by mean squared error
- Let X_i be a zero-one variable for $1 \le i \le N$. If you predict that $P(X_i = 1) = p_i$, your *Brier score* is

$$B = \frac{1}{N} \sum_{i=1}^{N} (p_i - X_i)^2 .$$

Low scores are good!

Glenn W. Brier

How confident should a point spread make us?

Probability and point spreads

- Our ratings measure points per game, not win probability.

# How confident should a point spread make us?

Probability and point spreads

- Our ratings measure points per game, not win probability.
- Sports books relate point spreads to moneyline odds/probability. [10]

ESPN BET NFL Odds - Week 12

Sunday, November 23

12:00 PM	Open	Spread	Total	ML
Indianapolis Colts (8-2)	o50.5 -110	+3.5 -115	o49.5 -115	+165
Kansas City Chiefs (5-5)	-3.5 -110	-3.5 -105	u49.5 -105	-195

12:00 PM	Open	Spread	Total	ML
New England Patriots (9-2)	-5.5 -115	-7 -105	o50.5 -120	-380
Cincinnati Bengals (3-7)	u50.5 -115	+7 -115	u50.5 EVEN	+290

12:00 PM	Open	Spread	Total	ML
Seattle Seahawks (7-3)	-12.5 -110	-13.5 -105	o40.5 -115	-1400
Tennessee Titans (1-9)	u42.5 -110	+13.5 -115	u40.5 -105	+750

How confident should a point spread make us?

Probability and point spreads

- Our ratings measure points per game, not win probability.
- Sports books relate point spreads to moneyline odds/probability. [10]
- Ratings
  $\Rightarrow$ point spreads
    $\Rightarrow$ probabilities
      $\Rightarrow$ Brier score

| Spread	p
0.0	0.5000
0.5	0.5174
1.0	0.5304
1.5	0.5395
2.0	0.5483
2.5	0.5638
3.0	0.5950
3.5	0.6270
4.0	0.6524
4.5	0.6656
5.0	0.6704
5.5	0.6784
6.0	0.6906
6.5	0.7073
7.0	0.7273
7.5	0.7421
8.0	0.7519
8.5	0.7642
9.0	0.7719
9.5	0.7787
10.0	0.7943
10.5	0.8088
11.0	0.8142
11.5	0.8266
12.0	0.8308
12.5	0.8336
13.0	0.8408
13.5	0.8519
14.0	0.8604

Brier scores by week — 2021–2025
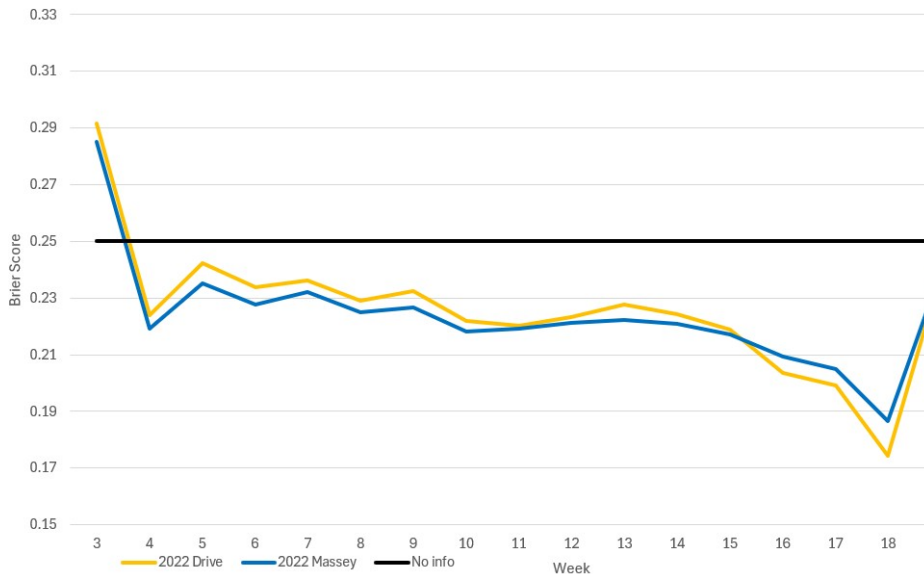

Brier Scores by Training Weeks (2021-2025)
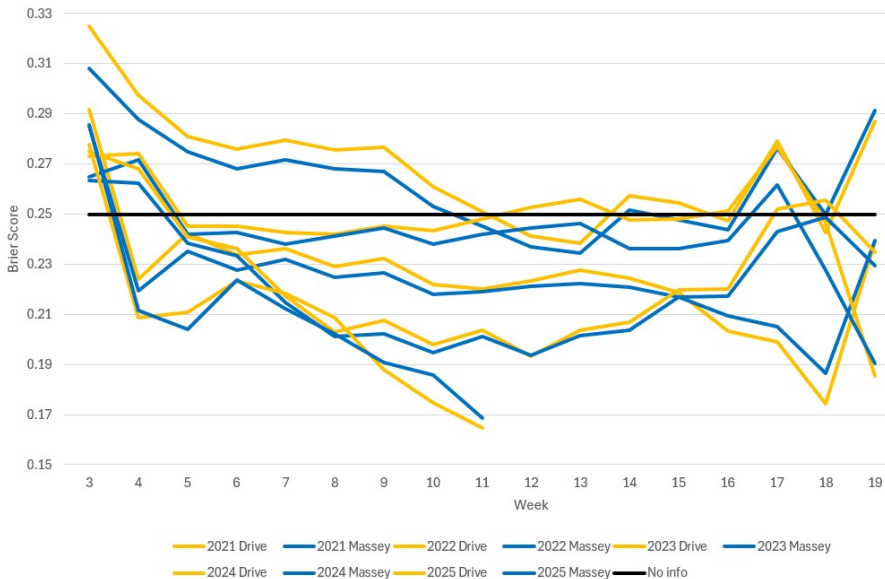
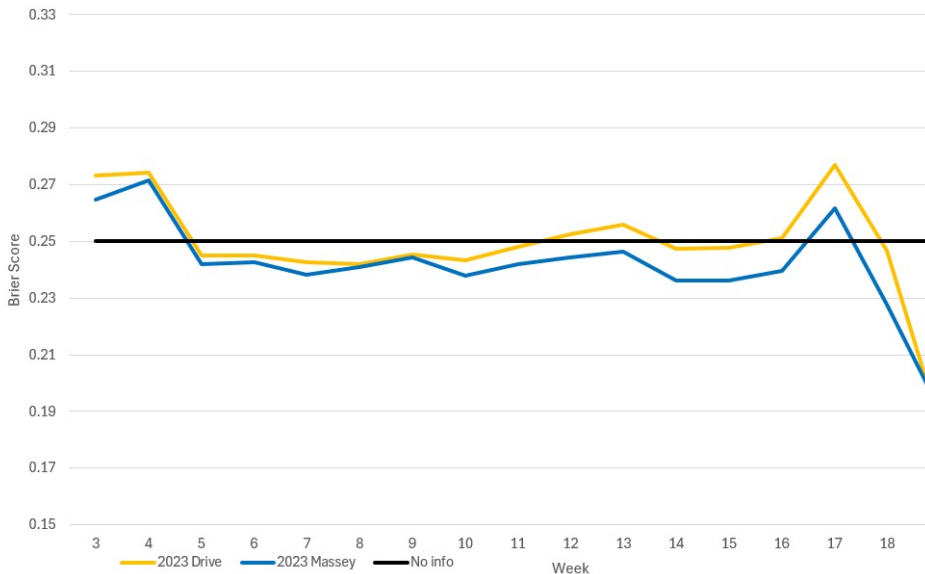# Brier scores by week — 2021–2025

Brier Scores by Training Weeks (2021)
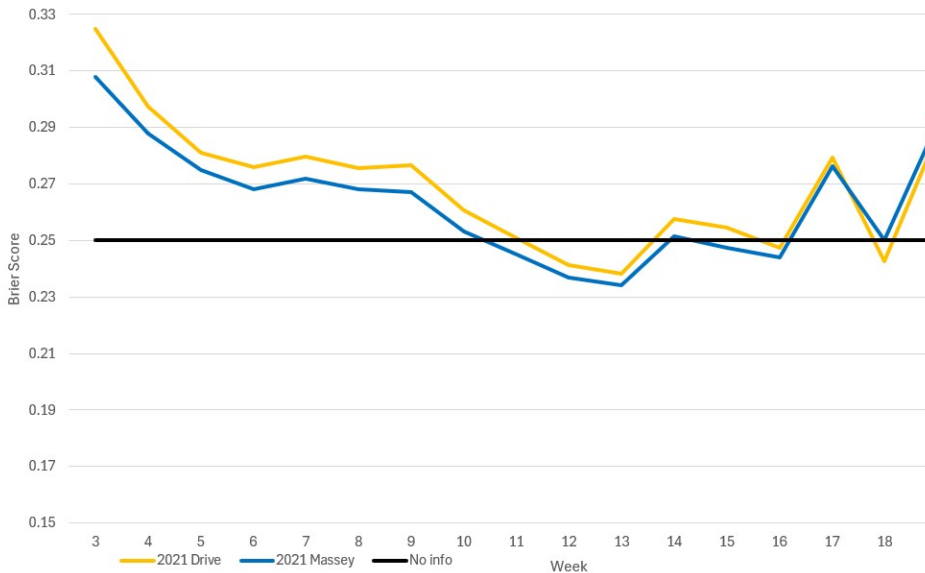
Brier scores by week — 2021–2025

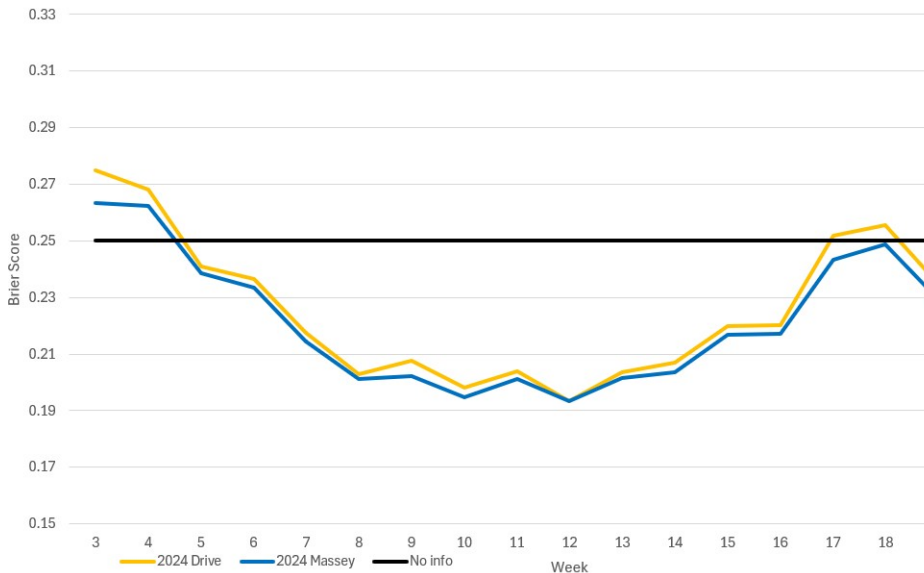Brier Scores by Training Weeks (2022)

Brier scores by week — 2021–2025

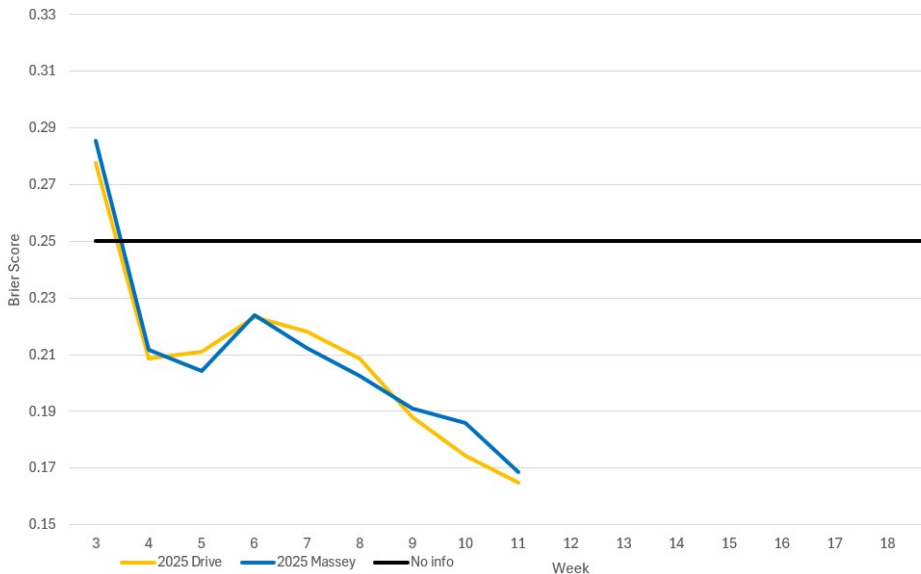Brier Scores by Training Weeks (2023)

Brier scores by week — 2021–2025

Brier Scores by Training Weeks (2024)

Brier scores by week — 2021–2025

Brier Scores by Training Weeks (2025)

Thanks!

Works Cited

[1] *Football - Composite Schedule - American Rivers Conference*, https://rollrivers.com/calendar.aspx?path=football. Accessed November 16, 2025.

[2] Ben Baldwin and Sebastian Carl, *nflfastR* (November 16, 2025), available at https://nflfastr.com/.

[3] Glenn W. Brier, *Verification of forecasts expressed in terms of probability*, Monthly Weather Review **78** (January 1950), no. 1, available at https://web.archive.org/web/20171023012737/https://docs.lib.noaa.gov/rescue/mwr/078/mwr-078-01-0001.pdf.

[4] Arthur Cayley, *A Memoir on the Theory of Matrices*, Philos. Trans. R. Soc. **148** (1858), available at https://jstor.org/stable/108649.

[5] Tim Chartier, *Mining the Ball Field* (April 5, 2012), https://www.huffpost.com/entry/mining-the-ball-field_b_1400696. Accessed November 16, 2025.

[6] Amy N. Langville and Carl D. Meyer, *Who's #1? The Science of Rating and Ranking*, Princeton University Press, 2013.

[7] Adrien Marie Legendre, *Nouvelles méthodes pour la détermination des orbites des comètes: avec un supplément contenant divers perfectionnemens de ces méthodes et leur application aux deux comètes de 1805* (1806).

[8] Kenneth Massey, *Statistical Models Applied to the Rating of Sports Teams* (Spring 1997), available at https://masseyratings.com/theory/massey97.pdf.

[9] Yves Nievergelt, *A tutorial history of least squares with applications to astronomy and geodesy*, Journal of Computational and Applied Mathematics **121** (2000), no. 1-2, 37–72.

[10] OddsJam, *Point Spread to Moneyline Converter — Spread vs Moneyline — OddsJam* (June 11, 2023), available at https://oddsjam.com/betting-calculators/point-spread.

[11] Hal S. Stern, *American Football*, Statistics in Sport. Chapter 1 (Jay Bennett, ed.), Arnold Applications of Statistics, Arnold, London, 1998, available at https://www.slideshare.net/slideshow/content-everywhere/6247354.

[12] Carl R. Yerger, *Nontraditional undergraduate research problems from sports analytics and related fields*, Involve **7** (2014), no. 3, 423–430, available at http://www.carlyerger.com/TURMStwo.pdf.

Thanks!

Image sources

- Glenn Brier. https://en.wikipedia.org/wiki/Glenn_W._Brier
- Tim Chartier. https://www.davidson.edu/people/tim-chartier
- Ben Collins. https://uwpexponent.com/news/2020/06/09/retiree-spotlight-thank-you-benjamin-collins/
- Crystal ball belonging to John Dee.
  https://commons.wikimedia.org/wiki/File:John_Dee's_crystal_ball_British_Museum_26_07_2013.jpg
- Division III playoff bracket. https://www.d3football.com/playoffs/2025/bracket
- Division III top 25. https://www.d3football.com/top25/2025/week11
- ESPN BET NFL Odds. https://www.espn.com/nfl/odds
- John Harris & Kevin Hutson. https://www.furman.edu/academics/mathematics/faculty/
- Amy N. Langville. https://charleston.edu/math/faculty-staff/faculty/langville-amy.php
- Kenneth Massey. https://www.cn.edu/people/kenneth-massey/
- Carl D. Meyer. https://math.sciences.ncsu.edu/people/meyer/
- Midwest Sports Analytics Meeting 2018. https://goodmanr.wixsite.com/mwsportsanalytics
- Hal Stern. https://www.tandfonline.com/doi/full/10.1080/09332480.2022.2145129
- Dan Swenson. https://bhsu.edu/directory/swenson_daniel.html
- *Who's #1?* book cover. https://www.amazon.com/Whos-1-Science-Rating-Ranking/dp/069116231X

Thanks!